GRAPHICAL PROJECTION OF THE DAILY ORBIT OF THE STAR SIRIUS ON THE PAVEMENT OF SAINT PETER'S SQUARE

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(Received 17 November 2022, revised 15 December 2022)

Abstract

In this paper we present geometrical projections of the daily orbit of the star Sirius on the pavement of Saint Peter's Square, in Vatican City. These projections are done - graphically and numerically - through certain sculptural motifs on top of the obelisk. These sculptural motifs reproduce the iconographic elements found on the coat of arms of Pope Alexander VII. We bring a new approach to the analysis of an internationally renowned square, which has great architectural and cultural heritage value. We make it clear that this research consists of a geometric analysis and not a historical study; that is why we show the results obtained without taking into account the intentionality or arbitrariness that Bernini's design entails.

Keywords: Saint Peter's Square, Caligula's obelisk, Star Sirius, oval

1. Introduction and preliminary remarks

This paper presents geometrical and graphical projections of the daily orbit $\mathcal{C}$ of the star *Sirius* (the brightest star in the night sky and possibly one of the most important in the history of humanity) on the pavement of Saint Peter's Square, in Vatican City. These projections are done through certain sculptural motifs on top of the obelisk located in the centre of the square - brought from Egypt by the emperor Caligula [1]. These sculptural motifs reproduce the iconographic elements found on the coat of arms of Pope Alexander VII. In order to determine the star position, we consider the period during which the square was designed by Gian Lorenzo Bernini (from 1665 to 1677, astronomical epoch *J1677*), under commission of Pope Alexander VII. This paper is a geometric analysis and not a historical survey.

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Since *Sirius* is the most visible star in the night sky and given the importance of the involved objects - both architectural and celestial - we believe it is of interest to present this paper (Figure 1). These projections of the orbit C result in several paths P_i on the square's pavement. By doing so, we bring a new approach to the analysis of an internationally renowned square which has great architectural and cultural heritage value. We make it clear that this research consists of a geometric analysis and not a historical study; that is why we show the results obtained without taking into account the intentionality or arbitrariness that Bernini's design entails.

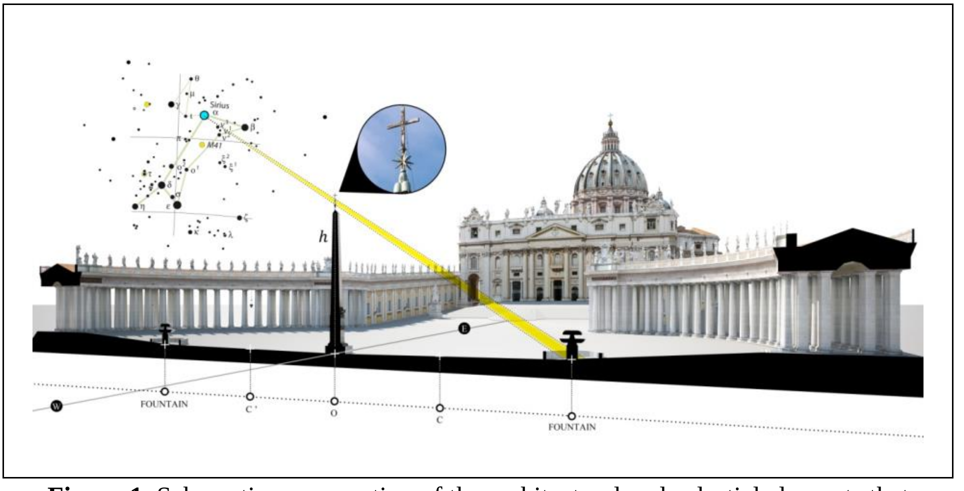

Figure 1. Schematic cross-section of the architectural and celestial elements that are used in the analysis (source: Authors).

1.1. Similar case in terms of procedure and iconography

For the whole procedure of projecting the orbit C and finding paths P_i on the pavement of Saint Peter's Square, we use numerical calculation.

1.1.1. Similarity as regards procedure

This process - projecting star orbits through a sculptural motif and particularly the orbit of the star *Sirius* - is not an original idea of ours nor it is attributable only to Saint Peter's Square central obelisk; it is an ancient process which was already used in some classical buildings [2]. A relevant example is the Clementine Gnomon in the Basilica of Santa Maria degli Angeli in Rome [3]. Pope Clement XI commissioned the astronomer and archaeologist Francesco Bianchini to create this Gnomon. It is an ingenious device that allows to see the star orbits during the day, and not only by night, projecting these daily orbits on the Basilica's floor (in the same way as we will do on the pavement of Saint Peter's Square).

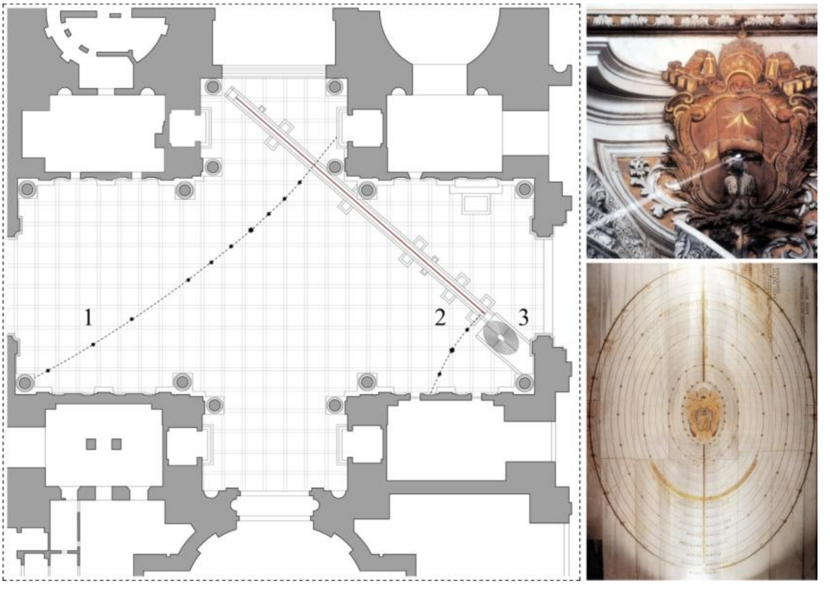

Figure 2. On the left, graphical representation showing the projections of the star orbits of Sirius 1, Arcturus 2 and Polaris 3 on the floor plan of Santa Maria degli Angeli e dei Martiri in Rome [3, p. 54]. On the right, a photo of the coat of arms of Pope Clement XI with its hole, and a photo of the orbit paths of Polaris on the floor plan of the church from astronomical epoch *J1700* to astronomical epoch *J2500* (source: Authors).

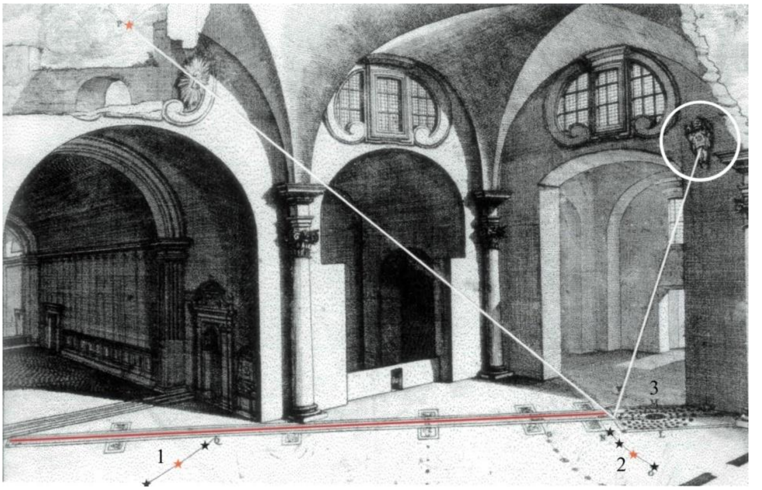

Figure 3. On the left, partial reproduction of a drawing showing how the star orbits of Sirius (1), Arcturus (2) and Polaris (3) are projected on the floor plan of Santa Maria degli Angeli e dei Martiri in Rome (source: Authors).

In fact, the Clementine Gnomon is a sundial, which is synchronized with sidereal time. Using this sundial it was possible to observe simultaneously the meridian transits of the Sun and the stars Sirius, Polaris and Arcturus (Figures 2 and 3), thanks to obtaining their daily paths as ellipse and hyperbola arcs on the Basilica's floor. In Figure 3 we present a drawing created by the authors based on a drawing that is in [3, p. 54].

These projections were made through holes in the ceiling and through a hole in the centre of the pope's coat of arms - which shows a star on its emblem - which is placed on a wall of the Basilica at a height of 20.34 meters (Figures 2 and 3). Similarly, in this paper we repeat the same process with sculptural motifs on the top of Saint Peter's Square central obelisk, numerically calculating the orbit of the star involved.

1.1.2. Similarity as regards iconography

Apart from the abovementioned similarity as regards the procedure used, there is also a similarity as regards iconography. The sculptural motifs on the top of the obelisk (excepting the cross) reproduce the graphic elements found on the coat of arms of Pope Alexander VII. Besides, there are clear similarities with the graphic elements found on the coat of arms of Pope Clement XI from the previous architectural example; Figure 4 shows these similarities.

In 1586 Pope Sixtus V decided to place the obelisk at its current location. When he did so, he removed the golden sphere, which used to top off the obelisk and he replaced it with a cross and some hills surmounted by an eight-pointed star (the arms of the Chigi family). Since then, many papal coats of arms display this iconography, notably the coat of arms of cardinal Fabio Chigi, who was elected as Pope Alexander VII after the 1565 conclave.

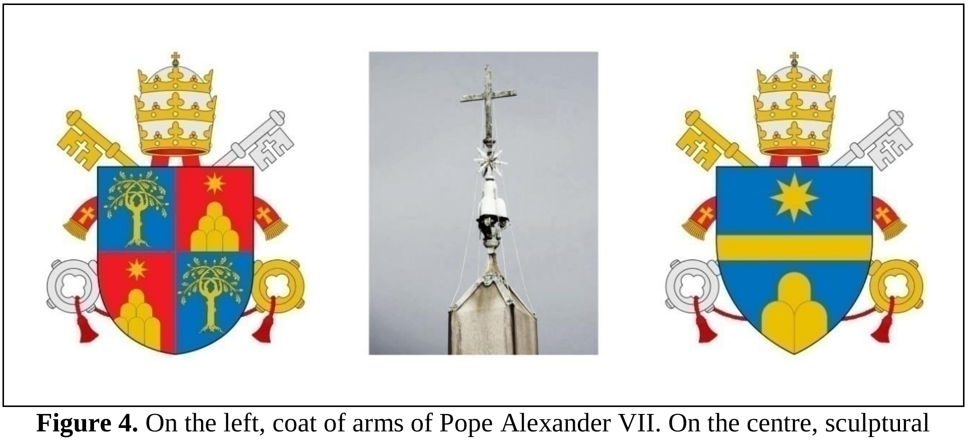

Figure 4. On the left, coat of arms of Pope Alexander VII. On the centre, sculptural motifs on top of Saint Peter's Square obelisk. On the right, coat of arms of Pope Clement XI (source: Authors).

1.2. About the Bernini's design for Saint Peter's Square

The following facts are well known: Bernini designed the square - from 1655 to 1667 - with a maximum diameter (major axis) of 198.12 m (887.23 Roman palms = 650 Roman feet) between the two colonnades. At the midpoint of such diameter we find Caligula's Egyptian obelisk, which marks the centre of the square. From a geometrical point of view, the square has the shape of a type-4 Serlian oval based on the *Vesica Piscis* [4, 5], the centres of which are clearly marked on the square's paving with a metal plaque bearing the words *centro del colonnato* [6-9] (see Figure 5 centre and Figure 7).

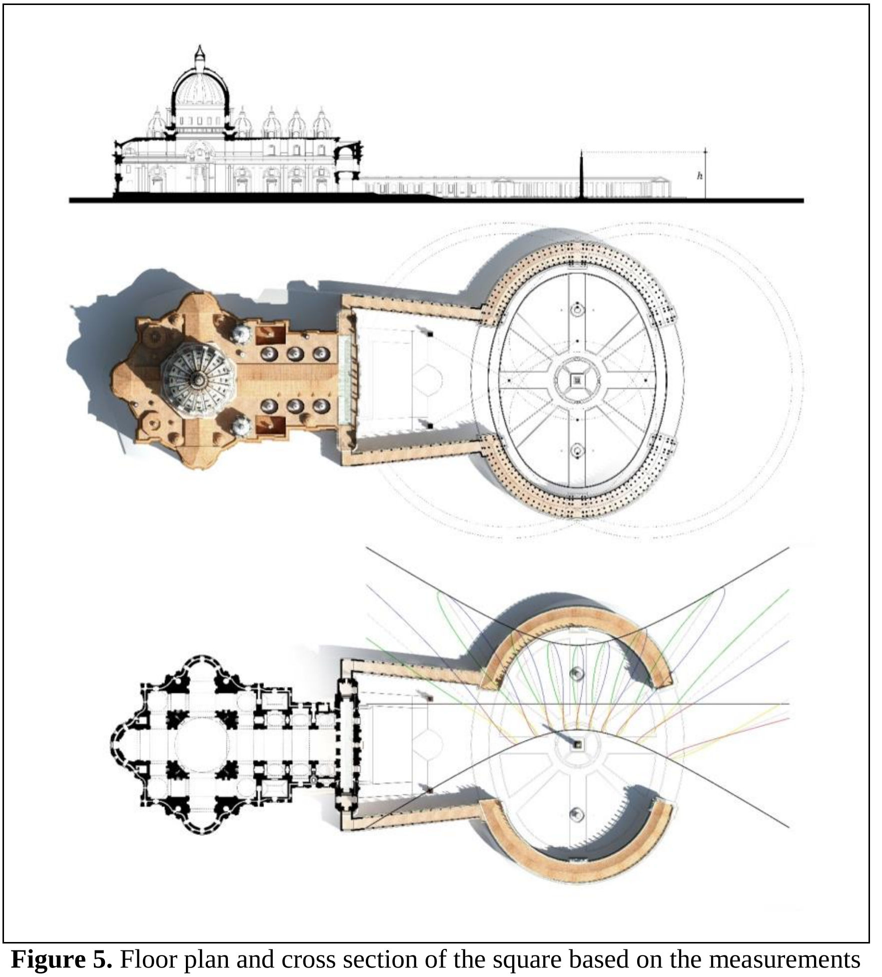

Figure 5. Floor plan and cross section of the square based on the measurements provided by the Vatican [10] and several onsite measurements taken by us, such as parameter *h*. The centre image shows the position of the fountains and the position of the foci of the ellipses, which are generated by the inner oval, and the outer oval (these foci are more clearly displayed in Figure 7). The lower image shows the analemmas generated by the tip of the obelisk (source: Authors).

For our paper, we have used data provided by the Vatican [10], several on-site measurements with a laser meter (Leica DISTO™ D510) and a verification graphic document created with a CAD-type vectorial software.

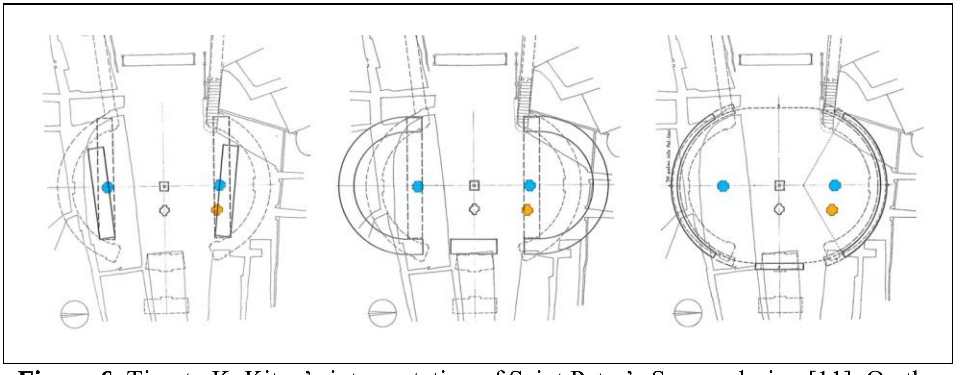

Figure 6. Timoty K. Kitao's interpretation of Saint Peter's Square design [11]. On the left, Bernini's trapezoid plan and proposed revision. On the centre, Bernini's first oval plan. On the right, Bernini's revised oval plan (arcades): key points and reconstruction. The position of the previous fountain (which was designed by Carlo Maderno) is highlighted in orange colour. The position of the present fountains (which were repositioned by Bernini) is highlighted in blue colour (source: Authors).

A previous fountain was designed and built by Carlo Maderno in 1613; the former position of this fountain is highlighted in orange colour in Figure 6. Bernini decided to reposition Maderno's fountain in 1677, aligning it with the obelisk on the major axis of the square's oval. The second fountain - which is identical to the first one, except that it bears the coat of arms of Pope Clement XI, while the first fountain is decorated with the coat of arms of Pope Paul V - was designed by Bernini and placed symmetrically to the first fountain with respect to the minor axis of the square's oval. The final positions of the two fountains are highlighted in blue colour in Figure 6 [11].

As for the geometry of the square's central obelisk, there are several papers concerning its role as a gnomon [12], i.e. as a solar shade caster. We have studied the shadow cast by this obelisk, and the results are shown in Figure 5 by means of the analemmas of the tip of the obelisk. The colours of the analemmas correspond to the four annual seasons. The centre straight line marks the equinoxes, and the outside hyperbolas mark the solstices. Of course, the analemmas (which are the paths of the shadow cast by a point at a given time throughout a year, using a mechanical clock to measure that time) have nothing to do with the times of Bernini, but we show them here for the purposes of illustration.

1.3. The foci of the ellipse that passes through the ends of the two axes of the oval determined by the shape of Saint Peter's Square

The following claim: "*the fountains position can be determined by means of the two foci of the ellipse passing through the ends of the two axes of the oval which shapes the square*" has been widely disseminated and it has even featured in scientific literature, for instance [12-14].

We are now going to show how accurate this claim is.

Let O be the square's centre, which marks the position of the obelisk (Figure 7). Let C and C' be the centres of the colonnade. Let $c = d(C,C')$ be the distance between these centres. Let $c/2 = d(O,C)$ be the distance between O and C; where c = 66.04 m = 295.74 *Roman palms*. Let G and G' be the two points which define the fountains positions. Let $d = d(O,G)$ be the distance between O and the fountains, where d = 59.43 m = 266.14 *Roman palms* (Figure 7).

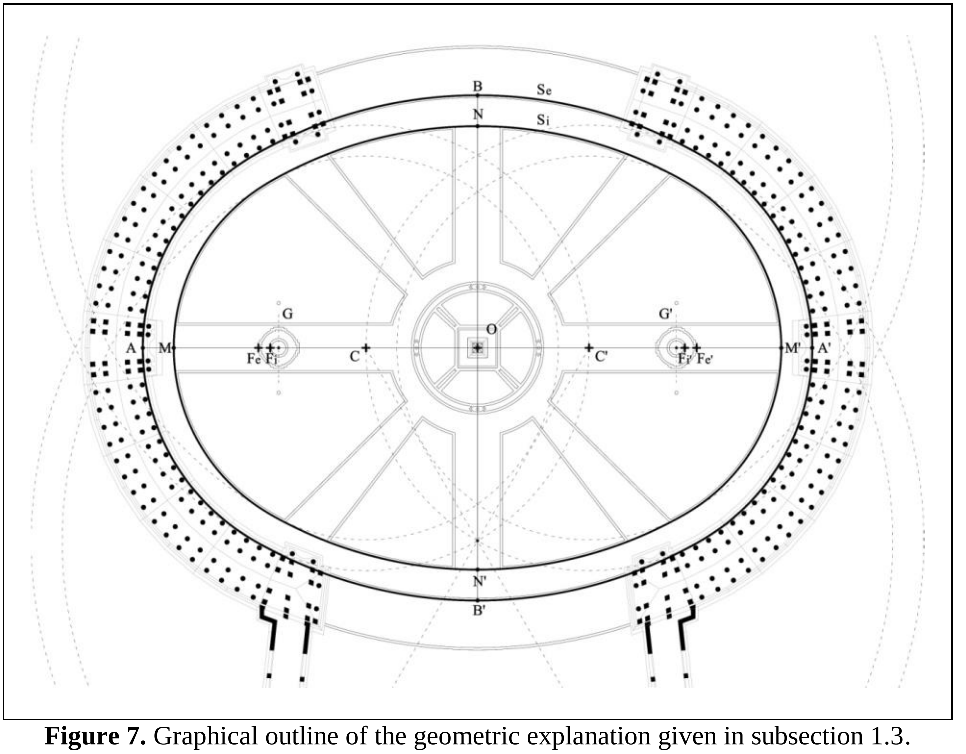

Figure 7. Graphical outline of the geometric explanation given in subsection 1.3. (source: Authors).

Using the fourth construction procedure for a Serlian oval, the outer oval is obtained from centres C and C'. We call this outer oval S_e. We know that 3c = 198.12 m = 887.23 *Roman palms*. The straight line CC' contains segment AA', which is the major axis of S_e; points A and A' are vertices of S_e. Therefore: $3c = d(A,A')$. After making the relevant geometric calculations, we find that segment BB' (which is the minor transverse axis of S_e [note that $d(B,B')$ is $2c + (2c-2h)$, where h is the height of the equilateral triangle with side length CC']) is such that $c*(4-\sqrt{3}) = d(B,B')$. Next, we consider the ellipse E_e, the vertices of which are A,A',B,B'. By means of calculation, we obtain the foci F_e and $F'_{e'}$ of this ellipse E_e; and we find that the distance between any of these foci and the obelisk is $d(O, F_e) = \sqrt{\frac{4\sqrt{3}-5}{2}} * c = 64.84\ m = 290.38$ *Roman palms* (Figure 7).

Hereinafter, the inner oval is called S_i. After taking measures at the square, we know that the straight line CC'' contains segment MM'', which is the major axis of S_i; and points M and M' are vertices of S_i, such that $2n = d(M,M')$ = 181.51 m = 812.85 *Roman palms*. After making the relevant geometric calculations, we find that segment NN'''' (which is the minor transverse axis of S_i [note that $d(N,N')$ is $(c+r) + ((c+r) - 2h)$, where h is the height of the equilateral triangle with side length CC', and $2r + c = 2n$, where r is the radius of the circles which determine oval S_i]) is such that $c*(1-\sqrt{3}) + 2n = d(N,N')$. Next, we consider the ellipse E_i, the vertices of which are M,M'',N,N''. By means of calculation, we obtain the foci F'_i and F_i' of this ellipse E_i; and we find that the distance between any these foci and the obelisk is $d(O,F_i) = \frac{\sqrt{3}-2}{2} * c^2 - n * (1-\sqrt{3}) * c = 61.91\ m = 277.24$ *Roman palms* (Figure 7).

On the basis of the above, we see that neither the distance between the obelisk and the focus of the outer ellipse $d(O,F_e)$ nor the distance between the obelisk and the focus of the inner ellipse $d(O,F_i)$ coincide with the distance between the obelisk and the fountains $d(O,G)$. With these calculations, in Table 2 we show how accurate is the claim mentioned at the beginning of this subsection.

2. Parameters to project the daily orbit of the star *Sirius* on the pavement of Saint Peter's Square

2.1. Parameters of Caligula's obelisk

In order to calculate the projection of the *Sirius* daily orbit on the pavement of Saint Peter's Square, through the sculptural motifs located between the tip of the obelisk and the 8-pointed metal star, we first need to determine the architectural parameters of the obelisk $\{\lambda, \varphi, r\}$, Figures 1 and 8. These parameters are: obelisk's longitude $\lambda = 12^h\ 24'\ 26''$, obelisk's latitude $\varphi = 41^o\ 54'\ 08''$, height to the obelisk's tip r = 34.88 m and height to the centre of the metal star r = 38.65 m.

2.2. Astronomical epoch J1677

As for the star positions, there is little difference between the present epoch and the reference epoch *J2000*, that is, the epoch of Julian day *JD(2451545.0)*. Astronomical epoch *J2000* corresponds to the astronomical positions on January 1st at noon in Greenwich Mean Time in Gregorian year 2000. However, as a result of the Earth's precessional motion and each star's proper motion, we must consider the astronomical epoch *J1677* (the period during which the square was designed), that is, the epoch of Julian day *JD(2333569,25)*.

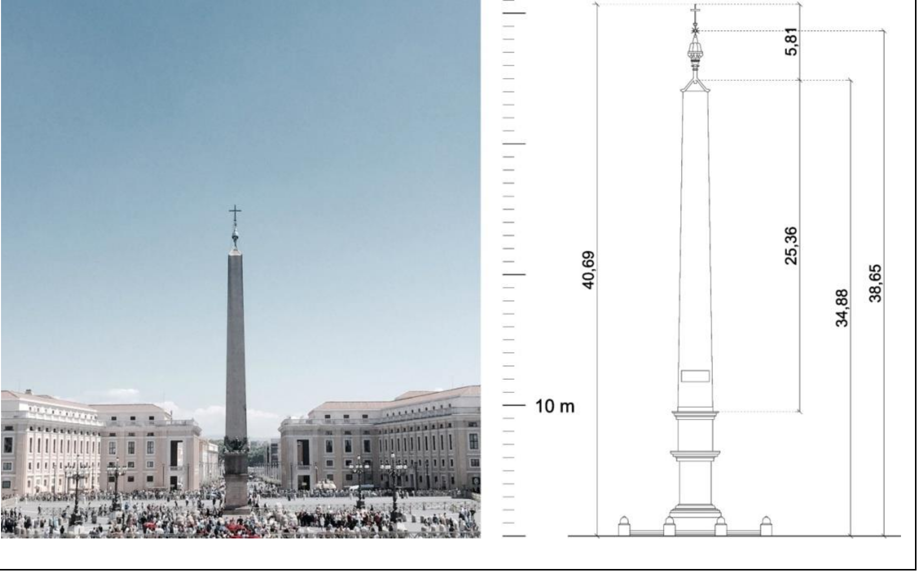

Figure 8. On the left, a picture of Caligula's obelisk, on the right, a sketch with the obelisk's dimensions (source: Authors).

Readers may turn to any book on Astronomy [15, 16] for definitions and calculations of: CE Gregorian year, CE and BCE Julian year, Julian day *JD(#)* according to the time measurement system proposed by Joseph Scaliger, and astronomical epoch *J#*.

In order to make the relevant calculations we have used the rigorous astronomical algorithms stated in [16] and the data contained in the SIMBAD database [http://simbad.u-strasbg.fr/simbad]. The results are shown in Table 1.

Table 1. Celestial coordinates of Sirius, the main star in Canis Majoris constellation.

Bayer	Star	Flux	Proper Motion		J2000.0		J1677.0	
Canis Majoris	Proper name	v	mas/yr in α	mas/yr in δ	R. Ascension α	Declination δ	R. Ascension α	Declination δ
α	*Sirius*	1.46	-546.01	-1223.07	$06^h 45^m 09^s$	-16º -42' -58"	$06^h 30^m 57^s$	-16º -18' -35"

3. Resulting graphic projections

After finding the position of *Sirius* on the celestial sphere during the period when the square was built, we will project the daily orbit of *Sirius* on the square's pavement through the sculptural motifs located on top of the obelisk. The final graphic result is displayed in Figure 9 and, in greater detail, in Figure 10.

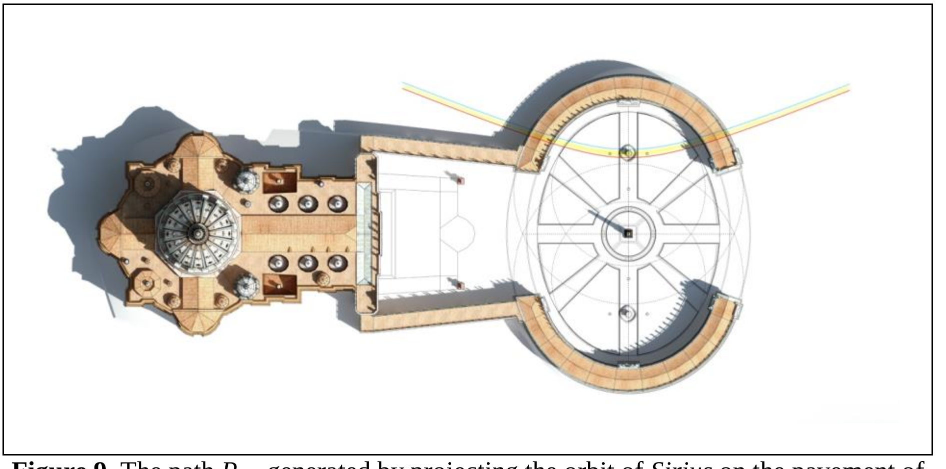

Figure 9. The path P_{red} generated by projecting the orbit of *Sirius* on the pavement of Saint Peter's Square through the tip of the obelisk is highlighted in red colour. The path P_{blue} generated by projecting the orbit of *Sirius* on the pavement of Saint Peter's Square through the metal star is highlighted in blue colour. The projection strip formed by all the paths P_i projected through the whole length of the papal sculptural motifs is highlighted in yellow (source: Authors).

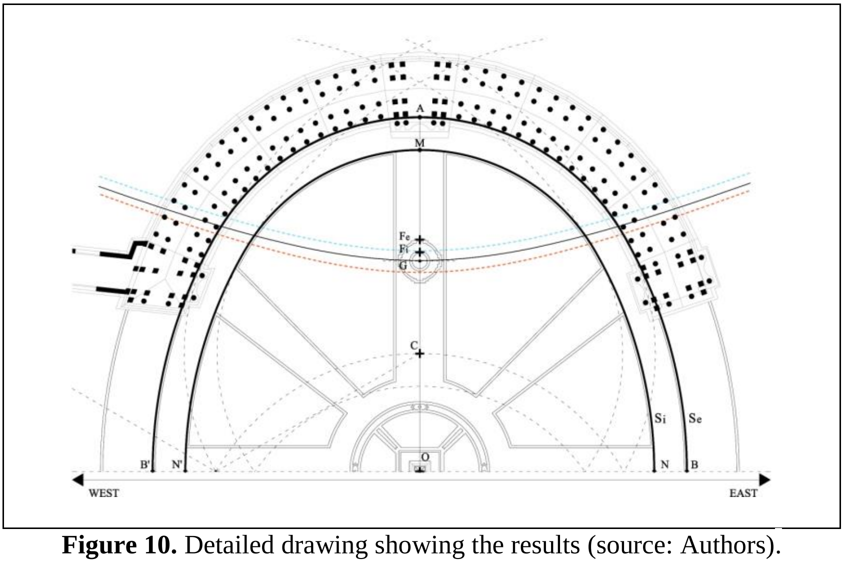

Figure 10. Detailed drawing showing the results (source: Authors).

In other words: each of the stars, and *Sirius* in particular, moves in a daily circular orbit *C* on the celestial sphere. With all parameters considered in section 2, the daily orbit *C* will be projected on the square's pavement through the sculptural motifs. These projections of the orbit *C* result in several paths P_i on the square's pavement. These paths P_i are conical curves (hyperbola arcs, in fact), as befits the projection of a circle on a plane. Yet we also need to calculate the position of circular circular *C* in epoch *J1677*. Some commercial computer programs provide the astronomical position of a star on the celestial sphere at any given moment, and even provide its relative position with respect to the horizon at a given place on Earth. However, we also need to project the circular daily orbits through a point selected by us and on a plane selected by us. On top of that, we want to have total control of the graphic process, entering all values in CAD vector format and precisely drawing the paths on the square's pavement. Therefore, we have created our own calculation software in C++ for the necessary astronomical algorithms [16] and the involved geometric and numerical methods. The results are displayed in a simple manner in Figures 9 and 10.

When projected through the tip of the obelisk, the daily orbit of *Sirius* generates a path P_{red} that we have highlighted in red colour. When projected through the metal star located on top of the obelisk, the daily orbit of Sirius generates a path P_{blue} that we have highlighted in blue colour. The projections passing through the sculptural motifs representing mountains (between the tip of the obelisk and the abovementioned metal star) generate a strip of paths that we have coloured in yellow.

4. Conclusions

By projecting the daily orbit of *Sirius* through the papal sculptural motifs on top of the obelisk in astronomical epoch *J1677* (when the square was designed), a projection strip is generated which passes through the position of the north fountain (*Fontana del Bernini, destra*) (Figures 9 and 10). By using the centre-point of the papal sculptural motifs, the projected path deviates 14 cm from the centre of the fountain (Table 2).

Table 2. Distances from the foci and from the projected paths to the centre of the fountain.

Distance	Meters	Roman palms
$d(F_e,G)$	5.41	24.23
$d(F_i,G)$	2.48	11.11
Metal star path: $d(P_{red},G)$	2.87	12.85
Obelisk tip path: $d(P_{blue},G)$	3.20	14.33
Centre path: $d(P_{black},G)$	0.14	1.79

The results of this paper are summarised in Table 2 and Figure 10.

We would like to highlight that we bring a new approach to the analysis of an internationally renowned square, which has great architectural and cultural heritage value. We make it clear that this research consists of a geometric analysis and not a historical study; that is why we show the results obtained without taking into account the intentionality or arbitrariness that Bernini's design entails.

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